Enrollment No: _____ Exam Seat No: _____

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Operations Research

Subject Code: 5SC01OPR1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 27/04/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions.** **(07)**

a. Which type of solution exist for the given LP problem. **02**

Maximize $Z = 3x_1 + 4x_2$

Subject to

$$x_1 - x_2 = -1$$
$$-x_1 + x_2 \leq 0$$

and $x_1, x_2 \geq 0$

b. If the optimal simplex table, $c_j - z_j = 0$ value indicates_____. **01**

c. Define: Slack variable. **01**

d. What is linear programming problem? **01**

e. Define: Convex Set. **01**

f. If dual has an unbounded solution, then primal has _____. **01**

Q-2 **Attempt all questions.** **(14)**

a. Explain the graphical method of solving an LP problem. Also solve the following LP problem by using graphical method. **07**

Maximize $Z = 3x_1 - 2x_2$

Subject to

$$x_1 + x_2 \leq 1$$
$$2x_1 + 2x_2 \geq 4$$

and $x_1, x_2 \geq 0$

b. Write the dual of the following primal LP problems. **04**

1. Maximize $Z = 2x_1 + 5x_2 + 6x_3$
   Subject to
   $$5x_1 + 6x_2 - x_3 \leq 3$$
   $$-2x_1 + x_2 + 4x_3 \leq 4$$
   $$x_1 - 5x_2 + 3x_3 \leq 1$$
   $$-3x_1 - 3x_2 + 7x_3 \leq 6$$
   and $x_1, x_2, x_3 \geq 0$
2. Minimize $Z = 3x_1 - 2x_2 + 4x_3$
   Subject to
   $$3x_1 + 5x_2 + 4x_3 \geq 7$$
   $$6x_1 + x_2 + 3x_3 \geq 4$$
   $$7x_1 - 2x_2 - x_3 \leq 10$$
   $$x_1 - 2x_2 + 5x_3 \geq 3$$
   $$4x_1 + 7x_2 - 2x_3 \geq 2$$
   and $x_1, x_2, x_3 \geq 0$

c. Define: Basic solution, Basic feasible solution, Optimum basic feasible solution. **03**

OR

Q-2 **Attempt all questions.** **(14)**

a. Use the simplex method to solve the following LP problem. **07**

Maximize $Z = 3x_1 + 5x_2 + 4x_3$
Subject to
$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
and $x_1, x_2, x_3 \geq 0$

b. A company produces two types of hats. Each hat of the first type requires twice as much labour time as the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second type of 150 and 250 hats. Assuming that the priority per hat are Rs. 8 for type A and Rs. 5 for type B, formulate the problem as a linear programming model in order to determine the number of hats to be produced of each type so as to maximize the profit. **04**

c. Prove that the dual of dual is primal. **03**

Q-3 **Attempt all questions.** **(14)**

a. Use the Big-M method to solve the following LP problem. **07**

Minimize $Z = 5x_1 + 3x_2$
Subject to
$$2x_1 + 4x_2 \leq 12$$
$$2x_1 + 2x_2 = 10$$
$$5x_1 + 2x_2 \geq 10$$
and $x_1, x_2 \geq 0$

b. The postal department is considering the purchase of vehicles to pick up and deliver mail from various offices. They are considering three types of vehicles. The cost of each of these are Rs. 5 lakhs, Rs. 10 lakhs and Rs. 8 lakhs per vehicle, respectively. These requires a crew of $2, 4$ and 4 persons per day considering multiple shifts. They expect these to run for $60, 100$ and 80 km per day. They expect that the total distance to be covered by the vehicles per day would be 2000km. Based on the fuel economy, the operating cost per day for these vehicles are Rs. 200, Rs. 350 and Rs. 300 per day. They have a budget restriction of Rs. 1.6 crore and have 80 people available as crew. Formulate a model to minimize the operating costs. **04**

c. Show that the following LP problem **03**

Maximize $Z = 4x_1 + 3x_2$

Subject to

$$x_1 - 6x_2 \leq 5$$
$$3x_1 \quad \leq 11$$

and $x_1, x_2 \geq 0$

has an unbounded solution.

OR

Q-3 **Attempt all questions.** **(14)**

a. Use two phase simplex method to solve the following LP problem. **07**

Maximize $Z = 5x_1 + 8x_2$

Subject to

$$3x_1 + 2x_2 \geq 3$$
$$x_1 + 4x_2 \geq 4$$
$$x_1 + x_2 \leq 5$$

and $x_1, x_2 \geq 0$

b. If $S = \{(x_1, x_2): x_1 \cdot x_2 \geq 1, x_1, x_2 \geq 0\}$, then show that S is convex set. **04**

c. Express the following LP problems in standard form. **03**

1. Maximize $Z = 5x_1 + 4x_2$

   Subject to

   $$6x_1 + 4x_2 \leq 24$$
   $$x_1 + 2x_2 \leq 6$$

   and $x_1, x_2 \geq 0$

2. Minimize $Z = x_1 - x_2 + x_3$

   Subject to

   $$x_1 + 2x_2 + x_3 \geq -1$$
   $$x_1 + x_2 + 2x_3 \geq 5$$

   and$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

SECTION – II

Q-4 **Attempt the following questions.** **(07)**

a. Write Kuhn-Tucker condition for maximization type non-linear program. **02**

b. Define: Feasible solution of a transportation problem. **01**

c. Which method gives best approximation to find out initial basic feasible solution of transportation problem? **01**

d. What is a necessary and sufficient condition for the existence of a feasible solution to the transportation problem? **01**

e. Write Hessian matrix of order three. **01**

f. What is unbalance assignment problem? **01**

Q-5 **Attempt all questions.** **(14)**

a. Explain the Modified distributive method for solving transportation problems. **07**

b. Solve the following non-linear programming problem using the method of Lagrange's multipliers. **07**

Maximize $Z = 5x_1 + x_2 - x_1^2 + 2x_1x_2 - x_2^2$

Subject to

$$x_1 + x_2 = 4$$

and$x_1, x_2 \geq 0$.

OR

Q-5 **Attempt all questions.** **(14)**

a. Explain the Hungarian method for solving assignment problems. **07**

b. Use the Kuhn-Tucker conditions to solve the following non-linear programming problem. **07**

Maximize $Z = 2x_1^2 - 7x_2^2 + 12x_1x_2$

Subject to

$$2x_1 + 5x_2 \leq 98$$

and$x_1, x_2 \geq 0$.

Q-6 **Attempt all questions.** **(14)**

a. Solve the following transportation problem using MODI Method. **07**

	D_1	D_2	D_3	Supply
O_1	04	03	02	10
O_2	02	05	00	13
O_3	03	08	06	12
Demand	08	05	04	

b. Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to each job is known and given in the following table **07**

		Jobs				
		I	II	III	IV	V
Men	A	2	9	2	7	1
	B	6	8	7	6	1
	C	4	6	5	3	1
	D	4	2	7	3	1
	E	5	3	9	5	1

Find the assignment of men to jobs that will minimize the total time taken.

OR

Q-6 **Attempt all questions.** **(14)**

a. Determine an initial basic feasible solution using (1) North-West corner method, (2) Vogel's approximation method, by considering the following transportation problem: **07**

	D_1	D_2	D_3	D_4	D_5	Supply
A	02	11	10	03	07	04
B	01	04	07	02	01	08
C	03	09	04	08	12	09
Demand	03	03	04	05	06	21

b. Use dual simplex method to solve the following LP problem. **07**

Minimize $Z = 3x_1 + x_2$

Subject to

$$x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 2$$

and $x_1, x_2 \geq 0$